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# Statement of Problem Studied and Results

Problems studied included existence and properties of solutions of nonlinear partial differential equations, including applications to shock waves and to differential geometry covering a very wide range of subjects. [4,7,10] developed variational approaches for an elliptic boundary value problem with nonlinear forcing term  $a(x)g(u)$ . The function  $a$  is allowed to change sign. New variational arguments to obtain positive solutions were introduced. [6] treats related problems using degree theory. In [5] and extended in [9] it was shown that for a constrained variational problem local  $C^1$  minima are also  $H^1$  local minima. This is used to obtain multiple solutions for a class of problems.

In [3] new results on existence and properties of principal eigenvalue for a general second order elliptic operator under Dirichlet conditions are obtained for arbitrary domains.

Gui studied semilinear heat equations and conformal metrics in  $\mathbb{R}^n$  as well as its Lotka-Volterra competition model. He also proved symmetry of the blowup set of porous-media-type equations.

Nabutovsky [20] used mathematical logic to study the space of triangulations of a compact manifold. A very surprising development, it connects with quantum gravity.

[21,22] deal with fluid dynamics vortex methods and entropy for hyperbolic conservation laws. Gui worked on problems for collisionless plasma.

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